

Geography of Fields in Extra Dimensions: String Theory Lessons for Particle Physics

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Abstract

String theoretical ideas might be relevant for particle physics model building. Ideally one would hope to find a unified theory of all fundamental interactions. There are only few consistent string theories in $D = 10$ or 11 space-time dimensions, but a huge landscape in $D = 4$. We have to explore this landscape to identify models that describe the known phenomena of particle physics. Properties of compactified six spatial dimensions are crucial in that respect. We postulate some useful rules to investigate this landscape and construct realistic models. We identify common properties of the successful models and formulate lessons for further model building.

1 Introduction

One of the main goals of string theory is the inclusion of the Standard Model (SM) of particle physics in an ultraviolet complete and consistent theory of quantum gravity. The hope is a unified theory of all fundamental interactions: gravity as well as strong and electroweak interactions within the $SU(3) \times SU(2) \times U(1)$ SM. Recent support for the validity of the particle physics Standard Model is the 2012 discovery of the “so-called” Higgs boson.

How does this fit into known string theory? Ideally one would have hoped to derive the Standard Model from string theory itself, but up to now such a program has not (yet) been successful. It does not seem that the SM appears as a prediction of string theory. In view of that we have to ask the question whether the SM can be embedded in string theory. If this is possible we could then scan the successful models and check specific properties that might originate from the nature of the underlying string theory.

Known superstring theories are formulated in $D = 10$ space time dimensions (or $D = 11$ for M theory) while the SM describes physics in $D = 4$. The connection between $D = 10$ and $D = 4$ requires the compactification of six spatial dimensions. The rather few superstring theories in $D = 10$ give rise to a plethora of theories in $D = 4$ with a wide spectrum of properties. The search for the SM and thus the field of so-called “String Phenomenology” boils down to a question of exploring this compactification process in detail.

But how should we proceed? As the top-down approach is not successful we should therefore analyse in detail the properties of the SM and then use a bottom-up approach to identify those regions of the “string landscape” where the SM is most likely to reside. This will provide us with a set of “rules” for $D = 4$ model constructions of string theory towards the constructions of models that resemble the SM.

The application of these rules will lead us to “fertile patches” of the string landscape with many explicit candidate models. Given these models we can then try to identify those properties of the models that make them successful. They teach us some lessons towards selecting the string theory in $D = 10$ as well as details of the process of compactification.

In the present paper we shall describe this approach to “string phenomenology”. In section 2 we shall start with “five golden rules” as they have been formulated some time ago [1]. These rules have been derived in a bottom-up approach exploiting the particular properties of quark- and lepton representations in the SM. They lead to some kind of (grand) unified picture favouring $SU(5)$ and $SO(10)$ symmetries in the ultraviolet. However, these rules are not to be understood as strict rules for string model building. You might violate them and still obtain some reasonable models. But, as we shall see, life is more easy if one follows these rules.

In section 3 we shall start explicit model building along these lines. We will select one of those string theories that allow for an easy incorporation of the rules within explicit solvable compactifications. This leads us to orbifold compactifications of the heterotic $E_8 \times E_8$ string theory [2, 3] as an example. We shall consider this example in detail and comment on generalizations and alternatives later. The search for realistic models starts with the analysis of the so-called \mathbb{Z}_6 -II orbifold [4, 5, 6, 7, 8, 9, 10, 11]. We define the search strategy in detail and present the results known as the “MiniLandscape” [8, 11], a fertile patch of the string landscape for realistic model building. We analyse the several hundred models of the MiniLandscape towards specific properties,

as e.g. the location of fields in extra-dimensional space. The emerging picture leads to a notion of “Local Grand Unification”, where some of the more problematic aspects of grand unification (GUT) can be avoided. We identify common properties of the successful models and formulate “lessons” from the MiniLandscape that should join the “rules” for realistic model building.

Section 4 will be devoted to the construction of new, explicit MSSM-like models using all \mathbb{Z}_N and certain $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifold geometries resulting in approximately 12000 orbifold models. Then, in section 5 we shall see how the lessons of the MiniLandscape will be tested in this more general “OrbifoldLandscape”.

In section 6 we shall discuss alternatives to orbifold compactifications, as well as model building outside the heterotic $E_8 \times E_8$ string. The aim is a unified picture of rules and lessons for successful string model building. Section 7 will be devoted to conclusions and outlook.

2 Five golden rules

Let us start with a review of the “Five golden rules for superstring phenomenology”, which can be seen as phenomenologically motivated guidelines to successful string model building [1]. The rules can be summarized as follows: we need

1. spinors of $SO(10)$ for SM matter
2. incomplete GUT multiplets for the Higgs pair
3. repetition of families from geometrical properties of the compactification space
4. $\mathcal{N} = 1$ supersymmetry
5. R -parity and other discrete symmetries

Let us explain the motivation for these rules in some detail in the following.

2.1 Rule I: Spinors of $SO(10)$ for SM Matter

It is a remarkable fact that the spinor **16** of $SO(10)$ is the unique irreducible representation that can incorporate exactly one complete generation of quarks and leptons, including the right-handed neutrino. Thereby, it can explain the absence of gauge-anomalies in the Standard Model for each generation separately. Furthermore, it offers a simple explanation for the observed ratios of the electric charges of all elementary particles. In addition, there is a strong theoretical motivation for Grand Unified Theories like $SO(10)$ from gauge coupling unification at the GUT scale $M_{\text{GUT}} \approx 3 \times 10^{16}$ GeV. Hence, the first golden rule for superstring phenomenology suggests to construct string models in such a way that at least some generations of quarks and leptons reside at a location in compact space, where they are subject to a larger gauge group, like $SO(10)$. Hence, these generations come as complete representations of that larger group, e.g. as **16** of $SO(10)$.

The heterotic string offers this possibility through the natural presence of the exceptional Lie group E_8 , which includes an $SO(10)$ subgroup and its spinor representation. Furthermore, using orbifold compactification the four-dimensional Standard Model gauge group can be enhanced to a local GUT, i.e. to a GUT group like $SO(10)$ which is realized locally at an orbifold singularity

in extra dimensions. In addition, there are matter fields (originating from the so-called twisted sectors of the orbifold) localised at these special points in extra dimensions and hence they appear as complete multiplets of the local GUT group, for example as **16**-plets of $\text{SO}(10)$.

On the other hand, the spinor of $\text{SO}(10)$ is absent in (perturbative) type II string theories, which can be seen as a drawback of these theories. Often this drawback manifests itself in an unwanted suppression of the top quark Yukawa coupling. On the other hand, F-theory (and M-theory) can cure this through the non-perturbative construction of exceptional Lie groups like e.g. E_6 . When two seven-branes with $\text{SO}(10)$ gauge group intersect in the extra dimensions, a local GUT can appear at the intersection. There, the gauge group can be enhanced to a local E_6 and a spinor of $\text{SO}(10)$ can appear as matter representation.

2.2 Rule II: Incomplete GUT Multiplets for the Higgs Pair

Beside complete spinor representations of $\text{SO}(10)$ for quarks and leptons, the (supersymmetric extension of the) Standard Model needs split, i.e. incomplete $\text{SO}(10)$ multiplets for the gauge bosons and the Higgs(-pair). Their unwanted components inside a full GUT multiplet would induce rapid proton decay and hence need to be ultra-heavy. In the case of the Higgs doublet, this problem is called the doublet-triplet splitting problem, because for the smallest GUT $\text{SU}(5)$ a Higgs field would reside in a five-dimensional representation of $\text{SU}(5)$, which includes beside the Higgs doublet an unwanted Higgs triplet of $\text{SU}(3)$. This problem might determine the localisations of the Higgs pair and of the gauge bosons in the compactification space: they need to reside at a place in extra dimensions where they feel the breaking of the higher-dimensional GUT to the 4D SM gauge group. Hence, incomplete GUT multiplets, e.g. for the Higgs, can appear. This is the content of the second golden rule.

In this way local GUTs exhibit grand unified gauge symmetries only at some special “local” surroundings in extra dimensions, while in 4D the GUT group seems to be broken down to the Standard Model gauge group. This allows us to profit from some of the nice properties of GUTs (like complete representations for matter as described in the first golden rule), while avoiding the problematic properties (like doublet-triplet splitting).

In the case of the heterotic string on orbifolds the so-called untwisted sector (i.e. the 10D bulk) can naturally provide such split $\text{SO}(10)$ multiplets for the gauge bosons and the Higgs. In particular, when the orbifold twist acts as a \mathbb{Z}_2 in one of the three complex extra dimensions, one can obtain an untwisted Higgs pair that is vector-like with respect to the full (i.e. observable and hidden) gauge group. Combined with an (approximate) R -symmetry this can yield a solution to the μ -problem of the MSSM. Furthermore, as all charged bulk fields originate from the 10D $E_8 \times E_8$ vector multiplet this scenario naturally yields gauge-Higgs-unification.

Finally, an untwisted Higgs pair in the framework of heterotic orbifolds can relate the top quark Yukawa coupling to the gauge coupling and hence give a nice explanation for the large difference between the masses of the third generation compared to the first and second one. In order to achieve this, the top quark needs to originate either from the bulk (as it is often the case in the MiniLandscape [8] of \mathbb{Z}_6 -II orbifolds) or from an appropriate fixed torus, i.e. a complex codimension one singularity in the extra dimensions.

2.3 Rule III: Repetition of Families

The triple repetition of quarks and leptons as three generations with the same gauge interactions but different masses is a curiosity within the Standard Model and asks for a deeper understanding. One approach from a bottom-up perspective is to engineer a so-called flavour symmetry: one introduces a (non-Abelian) symmetry group, discrete or gauge, and unifies the three generations of quarks and leptons in, for example, a single three-dimensional representation of that flavour group. However, as the Yukawa interactions violate the flavour symmetry, it must be broken spontaneously by the vacuum expectation value of some Standard Model singlet, the so-called flavon. This might explain the mass ratios and mixing patterns of quarks and leptons.

The third golden rule for superstring phenomenology asks for the origin of such a flavour symmetry. The rule suggests to choose the compactification space such that some of its geometrical properties lead to a repetition of families and hence yields a discrete flavour symmetry. In this case, the repetition of the family structure comes from topological properties of the compact manifold. Within the framework of type II string theories, the number of families can be related to intersection numbers of D-branes in extra dimensions, while for the heterotic string it can be due to a degeneracy between orbifold singularities. In the latter case, one can easily obtain non-Abelian flavour groups which originate from the discrete symmetry transformations that interchange the degenerate orbifold singularities, combined with a stringy selection rule that is related to the orbifold space group [12]. In any case the number of families will be given by geometrical and topological properties of the compact six-dimensional manifold.

2.4 Rule IV: $\mathcal{N} = 1$ Supersymmetry

Superstring theories are naturally equipped with $\mathcal{N} = 1$ or 2 supersymmetry in 10D. However, generically all supersymmetries are broken by the compactification to 4D. The fourth golden rule suggests to choose a “non-generic” compactification space such that $\mathcal{N} = 1$ survives in 4D. Examples for such special spaces are Calabi–Yaus, orbifolds and orientifolds. Motivation for this is a solution of the so-called “hierarchy problem” between the weak scale (a TeV) and the string (Planck) scale. Supersymmetry can stabilize this large hierarchy. Since such a supersymmetry appears naturally in string theory, we assume that $\mathcal{N} = 1$ supersymmetry will survive down to the TeV-scale.

2.5 Rule V: R -Parity and other Discrete Symmetries

Apart from the gauge symmetries of string theory, we need more symmetries to describe particle physics phenomena of the supersymmetric Standard Model. These could provide the desired textures of Yukawa couplings, explain the absence of flavour changing neutral currents, help to avoid too fast proton decay, provide a stable particle for cold dark matter and solve the so-called μ -problem. We know that (continuous) global symmetries might not be compatible with gravitational interactions. Hence, local discrete symmetries might play this role in string theory.

One of these symmetries is the well-known matter parity of the minimal supersymmetric extension of the Standard Model (MSSM): it forbids proton decay via dim. 4 operators and leads to a stable neutral WIMP candidate. Other discrete gauge symmetries are required to explain the flavour structure of quark/lepton masses and mixings.

3 The MiniLandscape

As we have seen in our review in section 2, the five golden rules [1] naturally ask for exceptional Lie groups. $SO(10)$, although it is not an exceptional group, fits very well in the chain of exceptional groups $E_8 \rightarrow E_7 \rightarrow E_6 \rightarrow SO(10) \rightarrow SU(5) \rightarrow SM$. Therefore, the $E_8 \times E_8$ heterotic string is the prime candidate and we choose it as our starting point. Alternatives to obtain E_8 in string theory are M- and F-theory, where such gauge groups can appear in non-perturbative constructions.

The implementation of the rules in string theory started with the consideration of orbifold compactifications of the $E_8 \times E_8$ heterotic string. This led to the so-called “heterotic brane world” [14] where toy examples have been constructed in the framework of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold. There, the explicit “geographical” properties of fields in extra dimensions have been presented and the local GUTs at the orbifold fixed points were analysed, see e.g. Fig. 1.

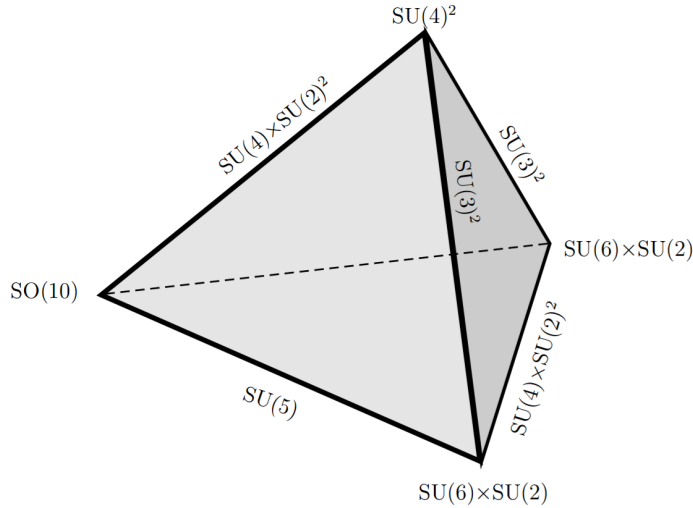


Figure 1: Gauge group topography from Ref. [13]. At different fixed points (corners of the tetrahedron), E_8 gets broken to different subgroups (U(1) factors are suppressed). At the edges we display the intersection of the two local gauge groups realised at the corners. The 4D gauge group is the standard model gauge group.

3.1 Exploring the \mathbb{Z}_6 -II orbifold

A first systematic attempt at realistic model constructions [8, 11] was based on the \mathbb{Z}_6 -II orbifold [4] of the $E_8 \times E_8$ heterotic string. This orbifold considers a six-torus defined by the six-dimensional lattice of $G_2 \times SU(3) \times SO(4)$ modded out by two twists, each acting in four of the six extra dimensions: θ of order 2 ($\theta^2 = 1$) and ω of order 3 ($\omega^3 = 1$), see Fig. 2.

In Ref. [8] the embedding of the twists into the $E_8 \times E_8$ gauge group was chosen in such a way that at an intermediate step there are local $SO(10)$ GUTs with localised **16**-plets for quarks and leptons. This choice can be motivated by rule I, as discussed in the previous section. Further breakdown of the gauge group to $SU(3) \times SU(2) \times U(1)$ is induced by two orbifold Wilson lines [15].

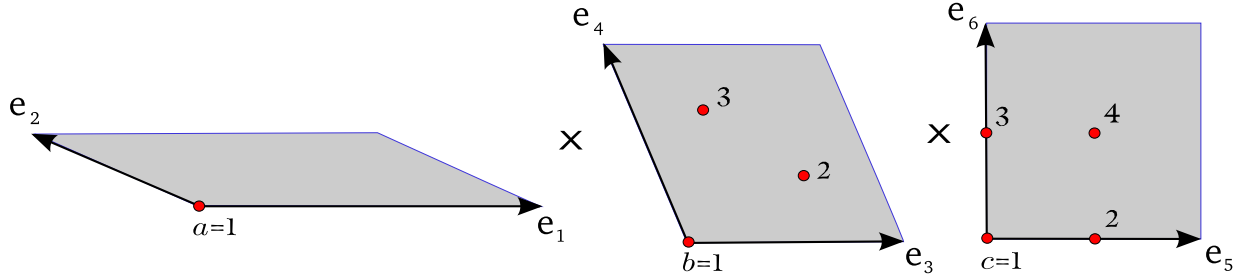


Figure 2: The six-dimensional torus (e_1, \dots, e_6) of the \mathbb{Z}_6 -II orbifold. In the θ -, ω -twisted sector the second, third torus is left invariant, respectively, while in the $\theta\omega$ -sector there are fixed points (labelled by a, b, c).

In this set-up, a scan for realistic models was performed using the following strategy:

- choose appropriate Wilson lines (and identify inequivalent models)
- SM gauge group $SU(3) \times SU(2) \times U(1)_Y \subset E_8$ times a hidden sector
- Hypercharge $U(1)_Y$ is non-anomalous and in $SU(5)$ GUT normalisation
- (net) number of three generations of quarks and leptons
- at least one Higgs pair
- exotics are vector-like w.r.t. the SM gauge group and can be decoupled

Using the above criteria, the computer assisted search led to a total of some 200 and 300 MSSM-like models in Refs. [8] and [11], respectively. The models typically have additional vector-like exotics as well as unbroken $U(1)$ gauge symmetries, one of which is anomalous. This anomaly induces an Fayet–Iliopoulos–term (FI-term), hence a breakdown of the additional $U(1)$ ’s and thus allows for a decoupling of the vector-like exotics. Explicit examples are given in Ref. [9] as benchmark models.

All fields of the models can be attributed to certain sectors with specific geometrical properties. In the present case there is an untwisted sector with fields in 10D (bulk), as well as twisted sectors where fields are localised at certain points (or two-tori) in the six-dimensional compactified space. The $\theta\omega$ twisted sector (Fig. 2) has fixed points and thus yields fields localised at these points in extra dimensions that can only propagate in our four-dimensional space–time. The θ and ω twisted sectors, in contrast, have fixed two-tori in extra dimensions. Fields in these sectors are confined to six space–time dimensions. Many properties of the models depend on these “geographic” properties of the fields in extra dimensions. For example, Yukawa couplings between matter and Higgs fields and in particular their coupling strengths are determined by the “overlap” of the fields in extra dimensions.

3.2 Lessons from the \mathbb{Z}_6 -II MiniLandscape

Given this large sample of realistic models, we can now analyse their properties and look for similarities and regularities. Which geometrical and geographical properties in extra dimensions are important for realistic models?

By construction, all the models have observable sector gauge group $SU(3) \times SU(2) \times U(1)$ and possibly some hidden sector gauge group relevant for supersymmetry breakdown. There is a net number of three generations of quarks and leptons and at least one pair of Higgs doublets H_u and H_d . The Higgs-triplets are removed and the doublet-triplet splitting problem is solved. A first question concerns a possible “ μ -term”: $\mu H_u H_d$ and we shall start our analysis with the Higgs-system, following the discussion of Ref. [16].

3.2.1 Lesson 1: Higgs-doublets from the bulk

The Higgs-system is vector-like and a μ -term $\mu H_u H_d$ is potentially allowed. As this is a term in the superpotential we would like to understand why μ is small compared to the GUT-scale: This is the so-called μ -problem. To avoid this problem one could invoke a symmetry that forbids the term. However, we know that μ has to be non-zero. Hence, the symmetry has to be broken and this might reintroduce the μ -problem again. In string theory the problem is often amplified since typically we find several (say N) Higgs doublet pairs. In the procedure to remove the vector-like exotics (as described above) we have to make $N - 1$ pairs heavy while keeping one light. In fact, in many cases the small μ -parameter is the result of a specific fine-tuning in such a way to remove all doublet pairs except for one. We do not consider this as a satisfactory solution. Fortunately, the models of the MiniLandscape are generically not of this kind.

Many MiniLandscape models provide one Higgs pair that resists all attempts to remove it. This is related to a discrete R-symmetry [9] that can protect the μ -parameter in the following way: In some cases the discrete R-symmetry is enlarged to an approximate $U(1)_R$ [17, 18].¹ Therefore, a μ -parameter is generated at a higher order M in the superpotential W , where the approximate $U(1)_R$ is broken to its exact discrete subgroup. This yields a suppression $\mu \sim \langle W \rangle \sim \epsilon^M$, where $\epsilon < 1$ is set by the FI parameter.

The crucial observation for this mechanism to work is the localisation of the Higgs pair H_u and H_d in agreement with our second golden rule: both reside in the 10D bulk originating from gauge fields in extra dimensions. Furthermore, the Higgs pair is vector-like with respect to all symmetries, gauge and discrete. This is related to the \mathbb{Z}_2 orbifold action in one of the two-tori. Hence, each term in the superpotential $f(\Phi_i) \subset W$ also couples to the Higgs pair, i.e. $f(\Phi_i) H_u H_d \subset W$. As SUSY breakdown requires a non-vanishing VEV of the superpotential the μ -term is related to the gravitino mass, i.e. $\mu = f(\langle \Phi_i \rangle) = \langle W \rangle \sim \epsilon^M \sim m_{3/2}$. This is a reminiscent of a field theoretical mechanism first discussed in Ref. [19].

3.2.2 Lesson 2: Top-quark from the bulk

Among all quarks and leptons the top-quark is very special: its large mass requires a large top-quark Yukawa coupling. Many MiniLandscape models address this naturally via the localisation of

¹In addition, $U(1)_R$ symmetries can explain vanishing vacuum energy in SUSY vacua.

the top-quark in extra dimensions: both (t, b) and \bar{t} reside in the 10D bulk, along with the Higgs pair. Hence, we have gauge-Yukawa unification and the trilinear Yukawa coupling of the top is given by the gauge coupling.

Typically the top-quark is the only matter field with trilinear Yukawa coupling. The location of the other fields of the third family is strongly model-dependent, but in general they are distributed over various sectors: the third family could be called a “patchwork family”.

3.2.3 Lesson 3: Flavour symmetry for the first two families

The first two families are found to be located at fixed points in extra dimensions (Fig. 2). As such they live at points of enhanced symmetries, both gauge and discrete.

The discrete symmetry is the reason for their suppressed Yukawa couplings. In the \mathbb{Z}_6 -II example shown in the figure two families live at adjacent fixed points in the third extra-dimensional torus: one family is located at $a = b = c = 1$, the other at $a = b = 1$ and $c = 3$ (see Fig. 2). Technically, this is a consequence of a vanishing Wilson line in the e_6 direction. This leads to a D_4 flavour symmetry [4, 12, 20]. The two localised families form a doublet, while the third family transforms in a one-dimensional representation of D_4 . This set-up forbids sizeable flavour changing neutral currents and thus relieves the so-called “flavour problem”. Furthermore, the geometric reason for small Yukawa couplings of the first and second family is their minimal overlap with the bulk Higgs fields. This leads to Yukawa couplings of higher order and a hierarchical generation of masses based on the Froggatt–Nielsen mechanism [21], where the FI-term provides a small parameter ϵ that controls the pattern of masses.

In addition, the first two families live at points of enhanced gauge symmetries and therefore build complete representations of the local grand unified gauge group, e.g. as **16**-plets of $SO(10)$. Hence, they enjoy the successful properties of “Local Grand Unification” outlined in the first golden rule.

3.2.4 Lesson 4: The pattern of SUSY breakdown

The question of supersymmetry breakdown is a complicated process and we shall try to extract some general lessons that are rather model-independent. Specifically we would consider gaugino condensation in the hidden sector [22, 23, 24, 25] realized explicitly in the MiniLandscape [26], see also section 5.4.

A reasonable value for the gravitino mass can be obtained if the dilaton is fixed at a realistic value $1/g^2(M_{\text{GUT}}) = \text{Re}S \approx 2$. Thus, the discussion needs the study of moduli stabilization, which, fortunately, we do not have to analyse here. In fact we can rely on some specific pattern of supersymmetry breaking which seems to be common in various string theories, first observed in the framework of Type IIB theory [27, 28, 29, 30, 31, 32, 33] and later confirmed in the heterotic case [34, 35]: so-called “mirage mediation”. Its source is a suppression of the tree level contribution in modulus mediation (in particular for gaugino masses and A-parameters). The suppression factor is given by the logarithm of the “hierarchy” $\log(M_{\text{Planck}}/m_{3/2})$, which numerically is of the order $4\pi^2$. Non-leading terms suppressed by loop factors can now compete with the tree-level contribution. In its simplest form the loop corrections are given by the corresponding β -functions, leading to

“anomaly mediation” if the tree level contribution is absent. Without going into detail, let us just summarise the main properties of mirage mediation:

- gaugino masses and A-parameters are suppressed compared to the gravitino mass by the factor $\log(M_{\text{Planck}}/m_{3/2})$
- we obtain a compressed pattern of gaugino masses (as the $SU(3)$ β -function is negative while those of $SU(2)$ and $U(1)$ are positive)
- soft scalar masses m_0 are more model-dependent. In general we would expect them to be as large as $m_{3/2}$ [29].

The models of the MiniLandscape inherit this generic picture. But they also teach us something new on the soft scalar masses, which results in lesson 4. The scalars reside in various localisations in the extra dimensions that feel SUSY in different ways: First, the untwisted sector is obtained from simple torus compactification of the 10D theory leading to extended $\mathcal{N} = 4$ supersymmetry in $D = 4$. Hence, soft terms of bulk fields are protected (at least at tree level) and broken by loop corrections when they communicate to sectors with less SUSY. Next, scalars localised on fixed tori feel a remnant $\mathcal{N} = 2$ SUSY and might be protected as well. Finally, fields localised at fixed points feel only $\mathcal{N} = 1$ SUSY and are not further protected [36, 37]. Therefore, we expect soft terms $m_0 \sim m_{3/2}$ for the localised first two families, while other (bulk) scalar fields, in particular the Higgs bosons and the stop, feel a protection from extended SUSY. Consequently, their soft masses are suppressed compared to $m_{3/2}$ (by a loop factor of order $1/4\pi^2$). This constitutes lesson 4 of the MiniLandscape.

4 The OrbifoldLandscape

The 10D heterotic string compactified on a six-dimensional toroidal orbifolds provides an easy and calculable framework for string phenomenology [2, 3]. A toroidal orbifold is constructed by a six-dimensional torus divided out by some of its discrete isometries, the so-called point group. For simplicity we assume this discrete symmetry to be Abelian. Combined with the condition on $\mathcal{N} = 1$ supersymmetry in 4D one is left with certain \mathbb{Z}_N and $\mathbb{Z}_N \times \mathbb{Z}_M$ groups, in total 17 different choices. For each choice, there are in general several inequivalent possibilities, e.g. related to the underlying six-torus. Recently, these possibilities have been classified using methods from crystallography, resulting in 138 inequivalent orbifold geometries with Abelian point group [38].

The orbifolder [39] is a powerful computer program to analyse these Abelian orbifold compactifications of the heterotic string. The program includes a routine to automatically generate a huge set of consistent (i.e. modular invariant and hence anomaly-free) orbifold models and to identify those that are phenomenologically interesting, e.g. that are MSSM-like.

A crucial step in this routine is the identification of inequivalent orbifold models in order to avoid an overcounting: even though the string theory input parameters of two models (i.e. so-called shifts and orbifold Wilson lines) might look different, the models can be equivalent and share, for example, the same massless spectrum and couplings. The current version (1.2) of the orbifolder uses simply the massless spectrum in terms of the representations under the full non-Abelian gauge group in order to identify inequivalent models. However, there are typically five to ten $U(1)$ factors

and the corresponding charges are neglected for this comparison of spectra, because they are highly dependent on the choice of $U(1)$ basis. As pointed out by Groot Nibbelink and Loukas [40] one can easily improve this by using in addition to the non-Abelian representations also the $U(1)_Y$ hypercharge as it is uniquely defined for a given MSSM model. We included this criterion into the orbifolder. However, it turns out that using this refined comparison method the number of inequivalent MSSM-like orbifold models increases only by 3%.

4.1 Search in the “OrbifoldLandscape”

Using the improved version of the orbifolder we performed a scan in the landscape of all \mathbb{Z}_N and certain $\mathbb{Z}_N \times \mathbb{Z}_M$ heterotic orbifold geometries for MSSM-like models, where our basic requirements for a model to be MSSM-like are:

- SM gauge group $SU(3) \times SU(2) \times U(1)_Y \subset E_8$ times a hidden sector
- Hypercharge $U(1)_Y$ is non-anomalous and in $SU(5)$ GUT normalisation
- (net) number of three generations of quarks and leptons
- at least one Higgs pair
- all exotics must be vector-like with respect to the SM gauge group

We identified approximately 12000 MSSM-like orbifold models that suit the above criteria. Given the large number of promising models we call them the “OrbifoldLandscape”. A summary of the results can be found in the appendix in Tabs. A.1 and A.2. Furthermore, the orbifolder input files needed to load these models into the program can be found at [41]. The scan did not reveal any MSSM-like models from orbifold geometries with \mathbb{Z}_3 , \mathbb{Z}_7 and $\mathbb{Z}_2 \times \mathbb{Z}_6$ -II point group. This is most likely related to the condition of $SU(5)$ GUT normalisation for hypercharge.

Note that this search for MSSM-like orbifold models is by far not complete. For example, we only used the standard $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifold geometries (i.e. those with label (1-1) following the nomenclature of Ref. [38]). In addition, our search was performed in a huge, but still finite parameter set of shifts and Wilson lines. Finally, the routine to identify inequivalent orbifold models can surely be improved further. Hence, presumably only a small fraction of the full heterotic orbifold Landscape has been analysed here.

4.2 Comparison to the Literature

Let us compare our findings to the literature. The \mathbb{Z}_6 -II (1-1) orbifold has been studied intensively in the past, see e.g. [4, 6, 7, 10]. Also the MiniLandscape [8, 11] was performed using this orbifold geometry, see section 3.1. In the first paper [8] local $SO(10)$ and E_6 GUTs were used as a search strategy and thus one was restricted to four out of 61 possible shifts, resulting in 223 MSSM-like models. In the second paper [11] this restriction was lifted, resulting in almost 300 MSSM-like models. They are all included in our set of 348 MSSM-like models from \mathbb{Z}_6 -II (1-1), see Tab. A.1 in the appendix.

Similar to \mathbb{Z}_6 -II, the $\mathbb{Z}_2 \times \mathbb{Z}_4$ orbifold geometry has been conjectured to be very promising for MSSM model-building [42]. Here, we can confirm this conjecture: we found 3632 MSSM-like

models from $\mathbb{Z}_2 \times \mathbb{Z}_4$ (1-1) — the largest set of models in our scan. Also from a geometrical point of view, the $\mathbb{Z}_2 \times \mathbb{Z}_4$ orbifold is very rich: there are in total 41 different orbifold geometries with $\mathbb{Z}_2 \times \mathbb{Z}_4$ point group, i.e. based on different six-tori and roto-translations [38]. We considered only the standard choice here, labelled (1-1). Hence, one can expect a huge landscape of MSSM-like models to be discovered from $\mathbb{Z}_2 \times \mathbb{Z}_4$.

Recently, Groot Nibbelink and Loukas performed a model scan in all \mathbb{Z}_8 -I and \mathbb{Z}_8 -II geometries [40]. They also used a local GUT search strategy (based on SU(5) and SO(10) local GUTs) and hence started with 120 and 108 inequivalent shifts for \mathbb{Z}_8 -I and \mathbb{Z}_8 -II, respectively. Their scan resulted in 753 MSSM-like models. Without imposing the local GUT strategy our search revealed in total 1713 MSSM-like models from \mathbb{Z}_8 , see Tab. A.1.

Further orbifold MSSM-like models have been constructed using the \mathbb{Z}_{12} -I orbifold geometry [43, 44]. This orbifold seems also to be very promising as we identified 750 MSSM-like models in this case, see Tab. A.1. Finally, we confirm the analysis of Ref. [45] for the orbifold geometries \mathbb{Z}_6 -I and \mathbb{Z}_N with $N = 3, 4, 7$ and standard lattice (1-1).

In the next section we will apply the strategies described by the “Five golden rules of superstring phenomenology” to our OrbifoldLandscape and search for common properties of our 12000 MSSM-like orbifold models. Thereby, we will see how many MSSM-like models would have been found following the “Five golden rules” strictly and how many would have been lost. Hence, we will estimate the prosperity of the “Five golden rules”.

5 Five golden rules in the OrbifoldLandscape

In the following we focus on the golden rules I - IV. A detailed analysis of rule V is very model-dependent and will thus not be discussed here.

5.1 Rule I: Spinor of SO(10) for SM matter

As discussed in Sec. 2.1 at least some generations of quarks and leptons might originate from spinors of SO(10) sitting at points in extra dimensions with local SO(10) GUT.² Hence, we perform a statistic on the number of such localisations in our 12000 MSSM-like orbifold models. The results are summarised in Tab. A.2 and displayed in Fig. 3.

It turns out that 25% of our models have at least one local SO(10) GUT. Furthermore, we find that some orbifolds seem to forbid local SO(10) GUTs with **16**-plets (for example \mathbb{Z}_6 -I [10]). On the other hand, the MSSM-like models from \mathbb{Z}_6 -II and \mathbb{Z}_8 -I (1-1) and (2-1) prefer zero or two localised **16**-plets of SO(10). Three local **16**-plets are very uncommon, they mostly appear in $\mathbb{Z}_2 \times \mathbb{Z}_4$.

Note that the number of local GUTs can be greater than three even though the model has a (net) number of three generations of quarks and leptons. Obviously, an anti-generation of quarks and leptons is needed in such a case. The maximal number we found in our scan is four local SO(10) GUTs with **16**-plets for matter in the cases of $\mathbb{Z}_2 \times \mathbb{Z}_2$ and $\mathbb{Z}_2 \times \mathbb{Z}_4$ orbifold geometries.

²See also [14, 5] and for an overview on local GUTs Ref. [10].

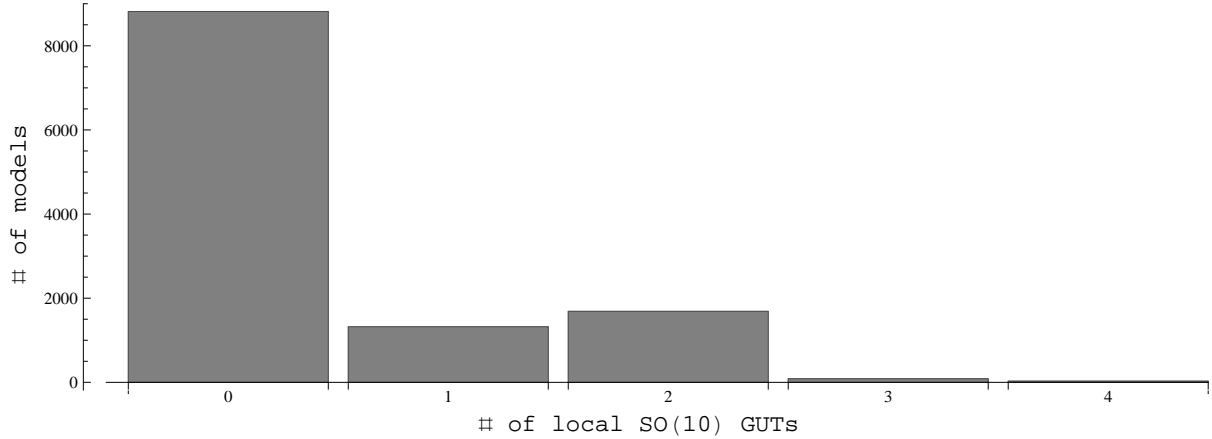


Figure 3: Number of MSSM-like orbifold models vs. number of local $\text{SO}(10)$ GUTs with **16**-plets for matter.

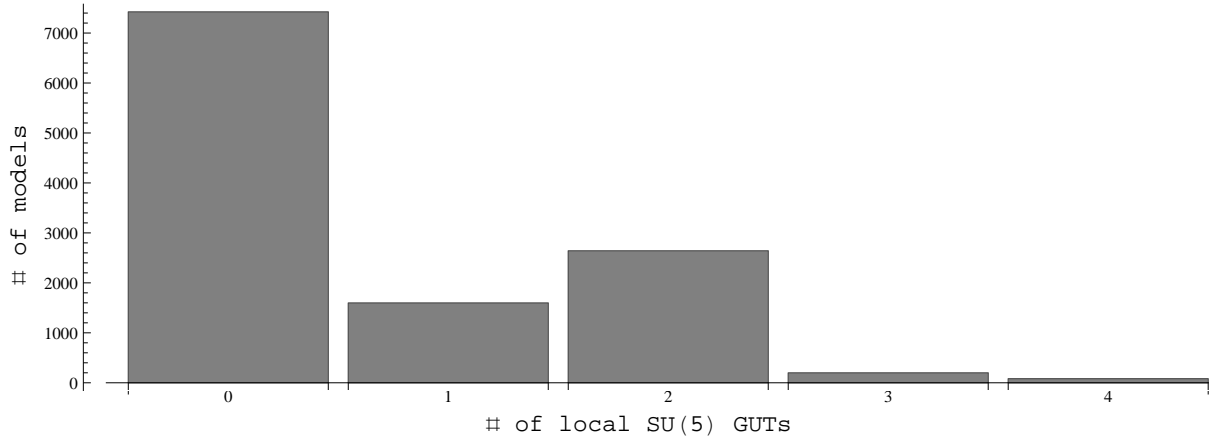


Figure 4: Number of MSSM-like orbifold models vs. number of local $\text{SU}(5)$ GUTs with **10**-plets for matter.

5.1.1 Other local GUTs

In addition, we analyse our 12000 models for local $\text{SU}(5)$ GUTs with local matter in **10**-plets. The results are summarised in Table A.2 and displayed in Fig. 4. We find this case to be very common: almost 40% of our MSSM-like models have at least one local **10**-plet of a local $\text{SU}(5)$ GUT.

Next, we also look for local E_6 GUTs with **27**-plets. We find only a few cases, most of them appear in $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifold geometries, see Table A.2.

Finally, we scan our models for localised SM generations (i.e. localised left-handed quark-doublets) transforming in a complete multiplet of any local GUT group that unifies the SM gauge group. Again, our results are listed in Table A.2 and visualised in Fig. 5. We find most of our models, i.e. 70%, have at least one local GUT with a localised SM generation.

In summary, the first golden rule, which demands for local GUTs in extra dimensions in order

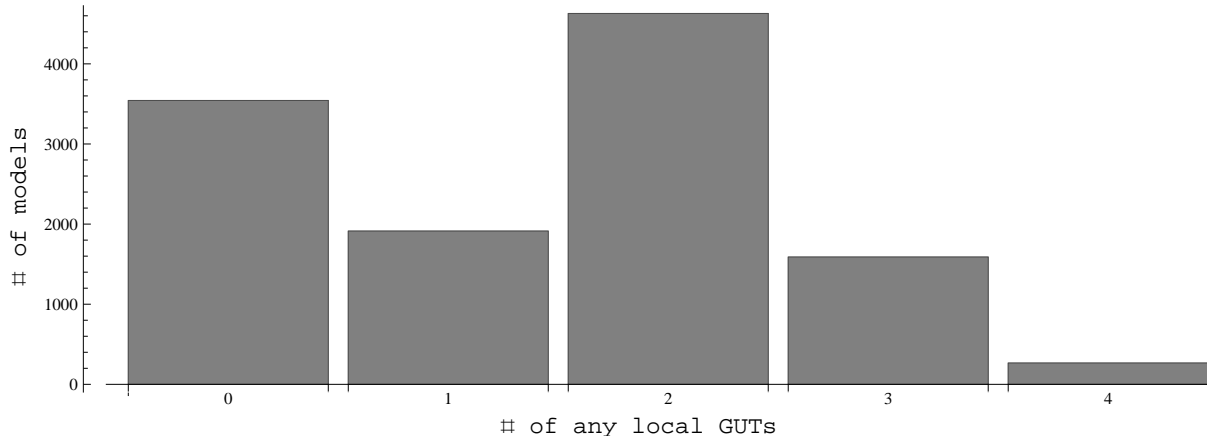


Figure 5: Number of MSSM-like orbifold models vs. number of local GUTs with local GUT multiplets for SM matter.

to obtain complete GUT multiplets for matter, is very successful: most of our 12000 MSSM-like models share this property automatically, it was not imposed by hand in our search.

5.2 Rule II: Incomplete GUT Multiplets for the Higgs Pair

Since the Higgs doublets reside in incomplete GUT multiplets, they might be localised at some region of the orbifold where the higher-dimensional GUT is broken to the 4D Standard Model gauge group. This scenario yields a natural solution to the doublet–triplet splitting problem. The untwisted sector (i.e. bulk) would be a prime candidate for such a localisation, but there can be further possibilities. The numbers of such GUT breaking localisations are summarised in Table A.1 and displayed in Fig. 6.

We see that GUT breaking localisations are very common among our MSSM-like models. Only a very few models do not contain any GUT breaking localisations that yield incomplete GUT multiplets for at least one Higgs. On the other hand, there are 4223 cases with one GUT breaking localisation — in most cases (4097 out of 4223) this is the bulk. In addition, there are many models that have more than one possibility for naturally split Higgs multiplets, but in almost all cases the bulk is among them.

Note that most of our MSSM-like models have additional exotic Higgs-like pairs, mostly two to six additional ones. In contrast to the MSSM Higgs pair they often originate from complete multiplets of some local GUT. On the other hand, we identified 1011 MSSM-like models with exactly one Higgs pair. Cases with exactly one Higgs pair, originating from the bulk might be especially interesting.

In summary, the second golden rule, which explains incomplete GUT multiplets for the Higgs using GUT breaking localisations in extra dimensions, is very successful — as in the case of the first golden rule, most of our 12000 MSSM-like models follow this rule automatically.

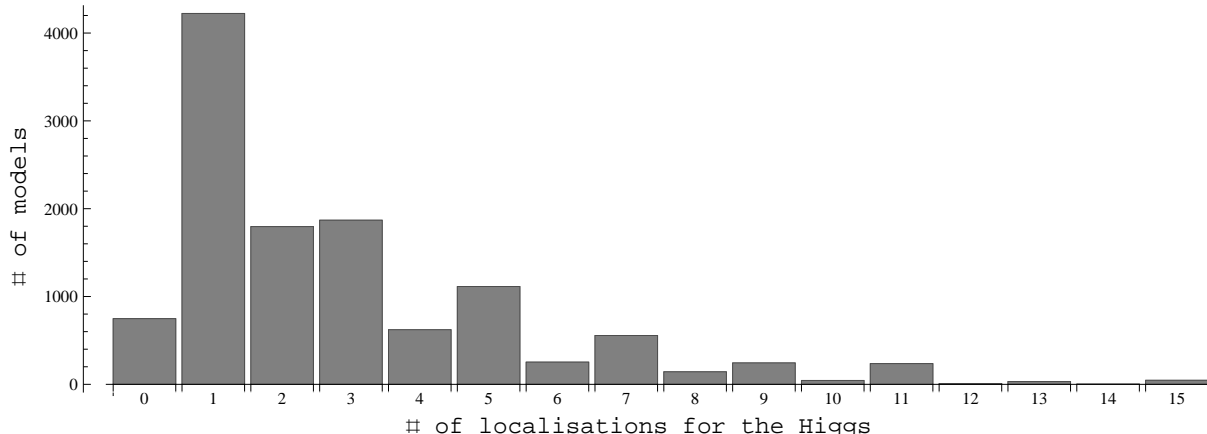


Figure 6: Number of orbifold models vs. number of localisations with broken local GUT such that only a Higgs doublet but not the triplet survives. The 10D bulk is the most common localisation of this kind.

5.3 Rule III: Repetition of families

The Standard Model contains three generations of quarks and leptons with a peculiar pattern of masses and mixings. This might be related to a (discrete) flavour symmetry.³

From the orbifold perspective discrete flavour symmetries naturally arise from the symmetries of the orbifold geometry [12, 20]. However, certain background fields (i.e. orbifold Wilson lines [15]) can break these symmetries. The maximal number of orbifold Wilson lines is six corresponding to the six directions of the compactified space. The orbifold-rotation, however, in general identifies some of those directions. Hence, the corresponding Wilson lines have to be equal. For example, the \mathbb{Z}_3 orbifold allows for maximally three independent Wilson lines.

In general, one can say that the more Wilson lines vanish the larger is the discrete flavour symmetry. On the other hand, non-vanishing Wilson lines are generically needed in order to obtain the Standard Model gauge group and to reduce the number of generations to three. Hence, it is interesting to perform a statistic on the number of vanishing Wilson lines for our 12000 MSSM-like orbifold models, see Tab. A.1 in the appendix and Fig. 7.

There are orbifold geometries, like \mathbb{Z}_4 , \mathbb{Z}_6 -I and \mathbb{Z}_{12} -I, apparently demanding for all possible orbifold Wilson lines to be non-trivial in order to yield the MSSM, see Tab. A.1. These MSSM-like models are expected to have no discrete, non-Abelian flavour symmetries. On the other hand, there are several orbifold geometries that seem to require at least one vanishing Wilson line in order to reproduce the MSSM with its three generations, for example \mathbb{Z}_6 -II, $\mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathbb{Z}_2 \times \mathbb{Z}_4$, $\mathbb{Z}_3 \times \mathbb{Z}_3$ and $\mathbb{Z}_4 \times \mathbb{Z}_4$. In general, the case of vanishing Wilson lines is very common: we see that in 75% of our MSSM-like orbifold models at least one allowed orbifold Wilson line is zero. In these cases non-Abelian flavour symmetries are expected. For example, most of the MSSM-like models from \mathbb{Z}_6 -II (1-1) have a D_4 flavour symmetry with the first two generations transforming as a doublet

³A gauged flavour symmetry like $SU(2)$ or $SU(3)$ is also possible. Some of the models in our OrbifoldLandscape realise this possibility, but we do not analyse these cases in detail here.

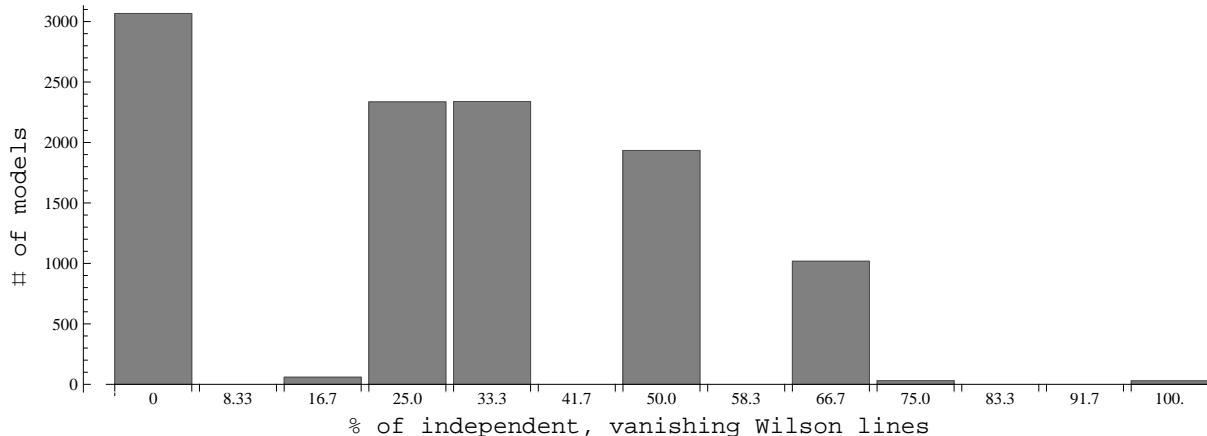


Figure 7: Number of MSSM-like orbifold models vs. the percentage of independent Wilson lines that are vanishing (e.g. the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold allows for six independent Wilson lines. If one is vanishing, the percentage is 16.7%). Higher percentages generically correspond to larger flavour symmetries.

and the third one as a singlet [4, 12, 20].

In summary, the third golden rule, which explains the origin of three generations of quarks and leptons by geometrical properties of the compactification space, is generically satisfied for our 12000 MSSM-like orbifold models.

5.4 Rule IV: $\mathcal{N} = 1$ supersymmetry

By construction, i.e. by choosing the appropriate orbifold geometries, our 12000 MSSM-like orbifold models preserve $\mathcal{N} = 1$ supersymmetry in four dimensions. This is expected to be broken by non-perturbative effects, i.e. by hidden sector gaugino condensation [22, 23, 24, 25]. Here, we follow the discussion of [26] where low energy supersymmetry breaking in the MiniLandscape of \mathbb{Z}_6 -II orbifolds was analysed. See also [46, 47] for a related discussion.

In detail, our MSSM-like models typically possess a non-Abelian hidden sector gauge group with little or no charged matter representations. The corresponding gauge coupling depends via the one-loop β -functions on the energy scale. If the coupling becomes strong at some (intermediate) energy scale Λ the respective gauginos condensate and supersymmetry is broken spontaneously by a non-vanishing dilaton F -term. Assuming that SUSY breaking is communicated to the observable sector via gravity the scale of soft SUSY breaking is given by the gravitino mass, i.e.

$$m_{3/2} \sim \frac{\Lambda^3}{M_{\text{Plank}}^2}, \quad (5.1)$$

where M_{Plank} denotes the Planck mass and the scale of gaugino condensation Λ is given by

$$\Lambda \sim M_{\text{GUT}} \exp \left(-\frac{1}{2\beta} \frac{1}{g^2(M_{\text{GUT}})} \right). \quad (5.2)$$

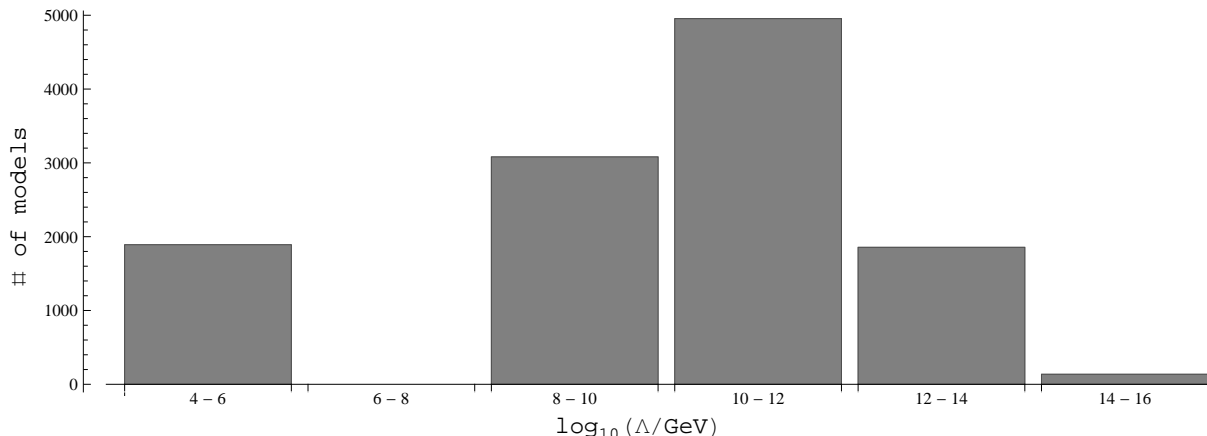


Figure 8: Number of MSSM-like orbifold models vs. scale of gaugino condensation for the largest hidden sector gauge group.

For every MSSM-like orbifold model we compute the β -function of the largest hidden sector gauge group under the assumption that any non-trivial hidden matter representation of this gauge group can be decoupled in a supersymmetric way. Furthermore, we assume dilaton stabilization at a realistic value $1/g^2(M_{\text{GUT}}) = \text{Re}S \approx 2$. Hence, we obtain the scale Λ of gaugino condensation. Our results are displayed in Fig. 8. For an intermediate scale $\Lambda \sim 10^{13}\text{GeV}$ one obtains a gravitino mass in the TeV range, which is of phenomenological interest.

The models in the OrbifoldLandscape seem to prefer low energy SUSY breaking. This result is strongly related to the heterotic orbifold construction: the $E_8 \times E_8$ gauge group in 10D is broken by orbifold shift and Wilson lines, which are highly constrained by string theory (i.e. modular invariance). Therefore, both E_8 factors get broken and not only the observable one. It turns out that the unbroken gauge group from the hidden E_8 has roughly the correct size to yield gaugino condensation at an intermediate scale and hence low energy SUSY breaking.

Note that our analysis is just a rough estimate as various effects have been neglected, for example the decoupling of hidden matter, the identification of the gaugino condensation and (string) threshold corrections. These effects can in principle affect the scale of SUSY breaking even by 2-3 orders of magnitude.

6 The general Landscape

With these considerations we have only scratched the surface of the parameter space of potentially realistic models. In addition, we have used “five golden rules” as a prejudice for model selection and it has to be seen whether this is really justified.

For general model building in the framework of (perturbative) string theory we have the following theories at our disposal:

- type I string with gauge group $\text{SO}(32)$
- heterotic $\text{SO}(32)$

- heterotic $E_8 \times E_8$
- type IIA and IIB orientifolds
- intersecting branes with gauge group $U(N)^M$

As we explained in detail, our rule I points towards exceptional groups and hence towards the $E_8 \times E_8$ heterotic string. On the other hand, type II orientifolds typically provide gauge groups of type $SO(M)$ or $U(N)$ and products thereof. Although we have $SO(2N)$ gauge groups in these schemes, matter fields do not come as spinors of $SO(2N)$, but originate from adjoint representations. In the intersecting brane models based on $U(N)^M$ gauge groups matter transforms in bifundamental representations of $U(N) \times U(L)$ (originating from the adjoint of $U(N+L)$). While this works nicely for the standard model representations, it appears to be difficult to describe a grand unified picture with e.g. gauge group $SU(5)$. Trying to obtain a GUT yields a gauge group at least as large as $U(5)$ and one has problems with a perturbative top-quark Yukawa coupling. One possible way out is the construction of string models without the prejudice for GUTs, see e.g. [48].

A comprehensive review on these intersecting brane model constructions can be found in the book of Ibáñez and Uranga [49] or other reviews [50]. These models have a very appealing geometric interpretation, see e.g. [51]: Fields are located on branes of various dimensions. Thus, physical properties of the models can be inferred from the localisation of the brane-fields in the extra dimensions and by the overlap of their wave functions, similar to the heterotic MiniLandscape. This nice geometrical set-up leads to attempts to construct so-called “local models”. Here, one assumes that all particle physics properties of the model are specified by some local properties at some specific point or sub-space of the compactified dimensions and that the “bulk” properties can be decoupled. However, the embedding of the local model into an ultraviolet complete and consistent string model is an assumption and its validity remains an open question.

Further schemes include “non-perturbative” string constructions:

- M-theory in $D = 11$
- heterotic M-theory $E_8 \times E_8$
- F-theory

These non-perturbative constructions are conjectured theories that generalize string theories or known supergravity field theories in higher dimensions. The low energy limit of M-theory is 11-dimensional supergravity. Heterotic M-theory is based on a $D = 11$ theory bounded by two $D = 10$ branes with gauge group E_8 on each boundary and F-theory is a generalization of type IIB theory, where certain symmetries can be understood geometrically. This non-perturbative construction allows for singularities in extra dimensions that lead to non-trivial gauge groups according to the so-called A-D-E classification. Groups of the A-type ($SU(N)$) and D-type ($SO(2N)$) can also be obtained in the perturbative constructions with D-branes and orientifold branes, while exceptional gauge groups can only appear through the presence of E-type singularities. This allows for spinors of $SO(10)$ and can produce a non-trivial top-quark Yukawa coupling within an $SU(5)$ grand unified theory. In that sense, F-theory can be understood as an attempt to incorporate rule I within type IIB theory. Unfortunately, it is difficult to control the full non-perturbative theory and the search

for realistic models is often based on local model building. Many questions are still open but there is enough room for optimism that promising models can be embedded in a consistent ultraviolet completion.

A general problem of string phenomenology is the difficulty to perform the explicit calculations needed to check the validity of the model. This is certainly true for the non-perturbative models, where we have (at best) some effective supergravity description. But also in the perturbative constructions we have to face this problem. We have to use simplified compactification schemes to be able to do the necessary calculations — we need a certain level of “Berechenbarkeit”. In our discussion we used the flat orbifold compactification that allows the use of conformal field theory methods. In principle, this enables us to do all the necessary calculations to check the models in detail. In the \mathbb{Z}_6 -II MiniLandscape this has been elaborated to a large extent. For the more general orbifold landscape, this still has to be done. Other constructions with full conformal field theory control are the free fermionic constructions [52] and the “tensoring” of conformal field theory building blocks: so-called Gepner models [53]. They share “Berechenbarkeit” with the flat orbifold models, but the geometric structure of compactified space is less transparent.

We have to hope that these simplified compactifications (or approximations) lead us to realistic models. In the generic situation one needs smooth manifolds, e.g. Calabi-Yau spaces, and some specific models have been constructed [54, 55]. However, these more generic compactifications require more sophisticated methods for computations that are only partially available, for example in order to determine Yukawa couplings. More recently a simplification based on the embedding of line bundles has allowed the constructions of many models [56, 57]. Still the calculational options are limited. It would be interesting to get a better geometric understanding of the compact manifold. At the moment the “determination” of couplings is based on a supergravity approximation using $U(1)$ symmetries. These symmetries are exact in this approximation at the “stability wall” but are expected to be broken to discrete symmetries in the full theory. This is in concord with rule V asking for the origin of discrete symmetries. Furthermore, this question has recently been analysed intensively within the various string constructions [58, 59, 60, 61, 62].

7 Summary

We have seen that there is still a long way to go in the search for realistic particle physics models from string theory. There are many possible roads but we are limited by our calculational techniques. Thus, in the near future we are still forced to make choices. Here, we have chosen to follow “five golden rules” outlined in section 2, which are mainly motivated by the quest for a unified picture of particle physics interactions. This strategy seems to require an underlying structure provided by exceptional groups pointing towards the $E_8 \times E_8$ heterotic string and F-theory.

Even given these rules, there are stumbling blocks because of the complexity of the compact manifolds. We cannot resolve these problems in full generality: we have to use simplified compactification schemes or approximations. We have to hope that nature has chosen a theory that is somewhat close to these simplified schemes. Of course, any method to go beyond this simplified assumptions should be seriously considered. However, there is some hope that this assumption might be justified: The orbifold models studied in this work have enhanced (discrete) symmetries that could be the origin of symmetries of the standard model, especially with respect to the fla-

your structure and symmetries relevant for proton stability as well as the absence of other rare processes. Generically, these symmetries are slightly broken as we go away from the orbifold-point. This gives rise to some hierarchical structures, for example for the ratio of quark masses in the spirit of Froggatt and Nielsen [21].

The analysis of the MiniLandscape can be seen as an attempt to study these questions in detail. Based on the availability of conformal field theory techniques we can go pretty far in the analysis of explicit models. A detailed analysis of the “OrbifoldLandscape” has not been performed yet, but should be possible along the same lines. In section 4 we started this enterprise of model building by constructing 12000 MSSM-like models. In a next step, the detailed properties of promising models have to be worked out. Especially the framework of the $\mathbb{Z}_2 \times \mathbb{Z}_4$ [42] should provide new insight into the properties of realistic models and might teach us further key properties shared by successful models.

One key property that we have learned is the geography of fields in the extra dimensions. The localisation of matter fields and the gauge group profiles in extra dimensions are essential for the properties of the low energy model. This is the first message of the heterotic orbifold construction and shared by the “braneworld” constructions in type II string theory and F-theory. Further lessons are:

- The Higgs pair is a bulk field. This allows for a convincing solution of the μ -problem using a (discrete) R-symmetry and yields doublet-triplet splitting.
- A sizeable value of the top-quark Yukawa coupling requires a sufficient overlap with the Higgs fields in extra dimensions. Thus, the top-quark should extend to the bulk as well.
- The matter fields of the first and second generation should be localised in a region of the extra-dimensional space where they are subject to an enhanced gauge symmetry, like $\text{SO}(10)$. This local GUT forces them to appear as complete representations, e.g. as spinors of $\text{SO}(10)$. Furthermore, the geometrical structure can manifest itself in a discrete flavour symmetry.
- The quest for low energy supersymmetry is the guiding principle in string model building. Still, it has to be seen whether this is realised in nature. At the moment no sign of supersymmetry has been found at the LHC, although the value of the Higgs mass is consistent with SUSY. The analysis of the models of the MiniLandscape and the location of the fields suggests a certain structure where even some remnants of extended supersymmetry (for fields in the bulk) seem to be at work. This picture of “heterotic supersymmetry” [36, 37] can hopefully be tested experimentally in the not too far future.

Acknowledgments

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A Summary of the OrbifoldLandscape

orbifold	# MSSM	max. # of indep. WLs	# models with					# models with							# MSSM without $U(1)_{\text{anom}}$
			0	1	2	3	≥ 4	0	1	2	3	4	5	≥ 6	
			indep. vanishing WLs					locations for split Higgs							
\mathbb{Z}_3 (1,1)	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0
\mathbb{Z}_4	(1,1)	0	4	0	0	0	0	0	0	0	0	0	0	0	0
	(2,1)	128	3	128	0	0	0	0	6	107	12	3	0	0	0
	(3,1)	25	2	25	0	0	0	0	0	25	0	0	0	0	0
\mathbb{Z}_6 -I	(1,1)	31	1	31	0	0	0	0	0	31	0	0	0	0	0
	(2,1)	31	1	31	0	0	0	0	0	31	0	0	0	0	0
\mathbb{Z}_6 -II	(1,1)	348	3	13	335	0	0	0	20	167	111	34	8	2	6
	(2,1)	338	3	10	328	0	0	0	19	162	107	33	9	2	6
	(3,1)	350	3	18	332	0	0	0	17	172	112	41	7	1	0
	(4,1)	334	2	39	295	0	0	0	17	161	113	32	11	0	0
\mathbb{Z}_7 (1,1)	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
\mathbb{Z}_8 -I	(1,1)	263	2	221	42	0	0	0	0	128	85	50	0	0	0
	(2,1)	164	2	123	41	0	0	0	0	76	53	35	0	0	0
	(3,1)	387	1	387	0	0	0	0	27	150	175	32	3	0	0
\mathbb{Z}_8 -II	(1,1)	638	3	212	404	22	0	0	12	257	165	123	16	50	15
	(2,1)	260	2	92	168	0	0	0	15	108	84	34	2	12	5
\mathbb{Z}_{12} -I	(1,1)	365	1	365	0	0	0	0	5	259	55	42	4	0	0
	(2,1)	385	1	385	0	0	0	0	7	271	63	44	0	0	0
\mathbb{Z}_{12} -II	(1,1)	211	2	135	76	0	0	0	9	40	107	31	12	4	8
$\mathbb{Z}_2 \times \mathbb{Z}_2$ (1,1)	101	6	0	59	42	0	0	0	79	0	10	3	8	0	1
$\mathbb{Z}_2 \times \mathbb{Z}_4$ (1,1)	3632	4	67	2336	1199	30	0	0	393	1194	160	690	83	449	663
$\mathbb{Z}_2 \times \mathbb{Z}_6$ -I	(1,1)	445	2	332	113	0	0	0	54	118	105	79	27	13	49
$\mathbb{Z}_2 \times \mathbb{Z}_6$ -II	(1,1)	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathbb{Z}_3 \times \mathbb{Z}_3$ (1,1)	445	3	1	369	75	0	0	0	27	212	1	15	102	0	88
$\mathbb{Z}_3 \times \mathbb{Z}_6$ (1,1)	465	1	441	24	0	0	0	0	4	39	64	82	88	110	78
$\mathbb{Z}_4 \times \mathbb{Z}_4$ (1,1)	1466	3	11	529	921	5	0	0	28	441	49	195	81	323	349
$\mathbb{Z}_6 \times \mathbb{Z}_6$ (1,1)	1128	0	1128	0	0	0	0	0	9	74	165	271	161	148	300
total	11940								748	4223	1796	1869	622	1114	1568
															102

Table A.1: Statistics on MSSM-like models (using the search criteria listed in Sec. 4.1) obtained from a random scan in all \mathbb{Z}_N and certain $\mathbb{Z}_N \times \mathbb{Z}_M$ heterotic orbifold geometries. The first column labels the geometry following the nomenclature from [38]. The second column gives the number of inequivalent MSSM-like models found in our scan. Next, we give the maximal number of independent Wilson lines (WLs) possible for the respective orbifold geometry and in the fourth column we count the number of MSSM-like models with a certain number (i.e. 0,1,2,3,4) of vanishing Wilson lines, see Sec. 5.3. In the fifth column we count the number of locations with broken local GUT such that Higgs-doublets without triplets appear, see Sec. 5.2. Finally, in the last column we give the number of models without $U(1)_{\text{anom}}$, i.e. without FI term.

orbifold		# models with					# models with			# models with					# models with				
		0	1	2	3	4	0	1	2	0	1	2	3	4	0	1	2	3	4
		local SO(10) GUTs					local E ₆ GUTs			local SU(5) GUTs					local GUTs				
\mathbb{Z}_3	(1,1)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
\mathbb{Z}_4	(1,1)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	(2,1)	78	50	0	0	0	50	78	0	128	0	0	0	0	0	128	0	0	0
	(3,1)	5	20	0	0	0	20	5	0	25	0	0	0	0	0	25	0	0	0
\mathbb{Z}_6 -I	(1,1)	31	0	0	0	0	31	0	0	31	0	0	0	0	31	0	0	0	0
	(2,1)	31	0	0	0	0	31	0	0	31	0	0	0	0	31	0	0	0	0
\mathbb{Z}_6 -II	(1,1)	155	2	187	4	0	332	6	10	203	12	133	0	0	2	3	293	4	46
	(2,1)	148	1	186	3	0	323	5	10	204	6	128	0	0	2	5	324	4	3
	(3,1)	164	1	185	0	0	328	11	11	202	12	136	0	0	2	11	293	9	35
	(4,1)	158	3	173	0	0	299	23	12	195	18	121	0	0	0	14	315	5	0
\mathbb{Z}_7	(1,1)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
\mathbb{Z}_8 -I	(1,1)	143	0	120	0	0	263	0	0	226	37	0	0	0	106	31	120	6	0
	(2,1)	92	0	72	0	0	164	0	0	147	17	0	0	0	75	15	74	0	0
	(3,1)	164	140	83	0	0	346	32	9	336	29	22	0	0	105	117	133	32	0
\mathbb{Z}_8 -II	(1,1)	428	77	133	0	0	638	0	0	276	155	207	0	0	79	194	355	10	0
	(2,1)	180	29	51	0	0	260	0	0	89	52	114	5	0	28	29	185	18	0
\mathbb{Z}_{12} -I	(1,1)	365	0	0	0	0	259	0	106	365	0	0	0	0	250	0	115	0	0
	(2,1)	385	0	0	0	0	269	0	116	385	0	0	0	0	269	0	116	0	0
\mathbb{Z}_{12} -II	(1,1)	110	69	32	0	0	177	31	3	86	78	47	0	0	0	80	131	0	0
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1,1)	72	6	12	1	10	66	33	2	75	0	11	0	15	3	18	8	30	42
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1,1)	2948	300	297	68	19	3181	358	93	2831	71	707	7	16	1918	70	670	911	63
$\mathbb{Z}_2 \times \mathbb{Z}_6$ -I	(1,1)	312	124	9	0	0	252	63	130	245	126	71	3	0	40	66	193	119	27
$\mathbb{Z}_2 \times \mathbb{Z}_6$ -II	(1,1)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1,1)	444	1	0	0	0	445	0	0	289	3	2	151	0	246	2	3	194	0
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1,1)	396	33	36	0	0	463	2	0	77	294	42	12	40	3	291	116	15	40
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1,1)	1246	116	94	10	0	1293	173	0	703	31	709	13	10	353	205	674	224	10
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1,1)	761	349	18	0	0	1122	6	0	274	656	191	7	0	0	609	511	8	0
total		8816	1321	1688	86	29	10612	826	502	7423	1597	2641	198	81	3543	1913	4629	1589	266

Table A.2: Statistics on MSSM-like models (using the search criteria listed in Sec. 4.1) obtained from a random scan in all \mathbb{Z}_N and certain $\mathbb{Z}_N \times \mathbb{Z}_M$ heterotic orbifold geometries. The first column labels the geometry following the nomenclature from [38]. The next four columns display the number of MSSM-like models with 0,1,2,3 and (up to) 4 local GUTs of specified gauge group with corresponding local matter: local SO(10) GUTs with local **16**-plets, local E₆ GUTs with local **27**-plets, local SU(5) GUTs with local **10**-plets and, finally, any local GUTs that unify SU(3) \times SU(2) \times U(1)_Y in a single gauge group with corresponding local matter representations containing left-handed quark doublets.

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